Differential Equations Eulers [30 marks]

1. [Maximum mark: 3] 26M.2.AHL.TZ3.12

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = y^3 - \frac{y}{x}$, for $x, y > 0$. It is given that $y = 2$ when $x = 1$.

(a) Use Euler's method with a step length of 0.01 to find an approximate value of y when $x = 1.05$. [3]

2. [Maximum mark: 4] 25N.2.AHL.TZ3.12

Consider the following homogenous differential equation

$\left(x^2 + xy\right)\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + xy - 3y^2$ where $x > 0$ and $y > \frac{x}{2}$.

It is given that $y = \frac{3}{2}$ when $x = 1$.

(a.i) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$. [1]

(a.ii) Use Euler's method with two equal steps to estimate the value of y when $x = 1.4$. [3]

3. [Maximum mark: 3] 25M.2.AHL.TZ1.9

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{x^2+y}$.

The solution curve passes through the point $(1,\ 0)$.

(a) Use Euler's method with a step value of 0.25 to estimate the value of y when $x = 2$. [3]

4. [Maximum mark: 3] 24M.2.AHL.TZ2.12

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} - y \operatorname{cosec} 2x = \sqrt{\tan x}$, where $0 < x < \frac{\pi}{2}$ and $y = \frac{\pi}{4}$ at $x = \frac{\pi}{4}$.

(a) Use Euler's method with step length $\frac{\pi}{12}$ to find an approximate value of y when $x = \frac{5\pi}{12}$.

Give your answer correct to three significant figures. [3]

5. [Maximum mark: 3] 23N.2.AHL.TZ1.9

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4-y}{10}$, where $y = 2$ when $x = 0$.

(a) Use Euler's method with a step size of 0.1 to find an approximation for y when $x = 0.5$. Give your answer correct to four significant figures. [3]

6. [Maximum mark: 14] 22M.2.AHL.TZ1.12

Consider the differential equation $x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - 2x^2$ for $x > 0$ and $y > 2x$. It is given that $y = 3$ when $x = 1$.

(a) Use Euler's method, with a step length of 0.1, to find an approximate value of y when $x = 1.5$. [4]

(b) By solving the differential equation, show that $y = \frac{8x + x^4}{4 - x^3}$. [10]

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